

Quantum evolution of the Universe from $\tau = 0$.

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We have shown that DeWitt constraint $H = 0$ on the physical states of the Universe does not prevent Heisenberg operators and its mean values to evolve with time. Mean value from observable, which is singular in classical theory, is also singular in a quantum case.

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I. INTRODUCTION

One of the first attempts to quantize gravity were made by DeWitt [1]. Then his equation $H\psi = 0$, where H is Hamiltonian, was applied for quantizing cosmology. In the simplest case of isotropic and uniform Universe filled with scalar field the equation contains two variables: scale factor of the Universe and amplitude of the scalar field. There is no "time" in this equation, whereas we are interested namely in the Universe evolution in time. This leads to various discussions about "time disappearance" and interpretation of wave function of Universe [2]. Possible solutions like to introduce time along quasi-classical trajectories, or subdivide Universe into classical and quantum parts were submitted [3]. Such point of view can not be satisfactory. Ideally, time must exist independently of we consider Universe quantum or classically. Let us remind that this situation is analogous to that in string theory, where constraint $H=0$ also exists. Nevertheless this constraint does not prevent evolution of Heisenberg operators $\hat{X}(\sigma, \tau)$ from τ .

Very short we can describe the problem as following. Let we have wave function $\psi(a, x)$ depending on two variables which satisfies

$$\hat{H}\psi(a, x) = 0. \quad (1)$$

For evolution of some Heisenberg operator \hat{H} we have

$$\langle A(\tau) \rangle = \langle \psi | e^{i\hat{H}\tau} \hat{A} e^{-i\hat{H}\tau} | \psi \rangle. \quad (2)$$

At first sight one sees no evolution, but it is not true for this case. Really $e^{-i\hat{H}\tau}|\psi\rangle = |\psi\rangle$, but we can't write $\langle \psi | e^{i\hat{H}\tau} = \langle \psi |$. The point is that the wave function can not be normalized the by an ordinary way $\int \psi^*(a, x) \psi(a, x) dx da = 1$ due to constraint. In fact the function is unrestricted along one of the variable. For instance, it is a variable. So if H contains differential operator like $\frac{\partial^2}{\partial a^2}$ we can not remove $\frac{\partial^2}{\partial a^2}$ to the left by habitual operation $\langle \psi | \frac{\partial^2}{\partial a^2} = \langle \frac{\partial^2}{\partial a^2} \psi |$ through the integration by parts. As a result, is not true to apply that $\langle \psi | \hat{H} = \langle \hat{H} \psi | = 0$ is valid.

Thus we need to solve equations for Heisenberg operators in quantum cosmology to obtain evolution of the Universe. In general case, solving of the equations for Heisenberg operators is hopeless problem, still, for the simplest case of two variables it be done.

II. RELATIVISTIC PARTICLE

The problem of quantization of cosmology have many similar features with that of the relativistic particle [4]. The action of the relativistic particle has the form:

$$S = -m \int \sqrt{-\dot{x}_\mu^2} d\tau. \quad (3)$$

The equivalent form, from which (3) can be obtained by the varying on the lapse function $e(\tau)$ is the

$$S = \frac{1}{2} \int (e^{-1} \dot{x}_\mu^2 - e m^2) d\tau. \quad (4)$$

One more equivalent form, from which (4) arises after varying p_μ , has the form:

$$S = \int \left\{ p_\mu \dot{x}^\mu - e \left(\frac{p_\mu^2 + m^2}{2} \right) \right\} d\tau. \quad (5)$$

Using the reparametrisation invariance [5] we can choose the lapse function equals to $e = 1/m$, so from the last equation one can see that Hamiltonian is equal to $H = \frac{p_\mu^2 + m^2}{2m}$ and, besides, varying on e gives constrain $H = 0$. After quantization

$$[\hat{p}^\mu, \hat{x}^\nu] = i g^{\mu\nu}, \quad \hat{p}^\mu = \{\hat{\varepsilon}, \hat{\mathbf{p}}\} \equiv \left\{ i \frac{\partial}{\partial t}, -i \nabla \right\}$$

the constrain becomes the Klein-Gordon equation. Commuting the position and four-momentum operators with the Hamiltonian one obtains Heisenberg equation of motion:

$$\begin{aligned} \frac{d\hat{x}^\mu}{d\tau} &= i[\hat{H}, \hat{x}^\mu] = \frac{\hat{p}^\mu}{m}, \\ \frac{d\hat{p}^\mu}{d\tau} &= 0 \end{aligned}$$

Evident solution of the equation of motion is

$$\begin{aligned} \hat{x}^\mu(\tau) &= x^\mu + \frac{p^\mu(0)}{m} \tau \\ \hat{p}^\mu(\tau) &= \hat{p}^\mu, \end{aligned}$$

where $\hat{x}(0) = x$ and $\hat{p}(0) = \hat{p}$. As we see two "times" appears in the above equations. We shall refer to the

$\hat{t}(\tau)$ as "physical time" and to the τ as a "proper time". Physical time is operator, whereas proper time is some parameter "always running forward" like time in Newtonian physics.

One can easily check that the wave function satisfying Klein-Gordon equation can't be normalized through the integration d^4x [3], however, it can be normalized through the time-like component of the conserved four-current:

$$\langle \psi | \psi \rangle = -i \int \{ \psi(\mathbf{r}, t) \partial_t \psi^*(\mathbf{r}, t) - (\partial_t \psi(\mathbf{r}, t)) \psi^*(\mathbf{r}, t) \} d^3\mathbf{r}.$$

Normalized wave packets have the form

$$\psi(x) = \int \frac{a(\mathbf{k}) e^{-i\varepsilon(\mathbf{k})t + i\mathbf{k}\mathbf{r}}}{\sqrt{2\varepsilon(\mathbf{k})(2\pi)^3}} d^3\mathbf{k}; \quad \int |a(\mathbf{k})|^2 d^3\mathbf{k} = 1,$$

where $\varepsilon(\mathbf{k}) = \mathbf{k}^2 + m^2$. To avoid appearance of the states with negative norm we must take only positive frequency solutions. Using this wave packet we see that integration $\int \psi^*(x) \psi(x) d^4x$ does not converge and conclude that the wave function is not restricted over the t variable. As a consequence, Heisenberg operators evolve with τ despite of constrain $H\psi = 0$.

Define now mean value of some operator A as

$$\begin{aligned} \langle \hat{A}(\tau) \rangle = & i \int \left(\frac{\partial \psi}{\partial t} (\hat{A}(\tau) \psi)^* \right. \\ & \left. - \left(\frac{\partial \psi^*}{\partial t} \right) \hat{A}(\tau) \psi \right) d^3\mathbf{r} \Big|_{t=0}. \end{aligned} \quad (6)$$

Let us note, that after integration over $d^3\mathbf{r}$ in (6) we should set $t = 0$. This definition has the following properties:

- 1) It is consistent with the normalization of wave function if we choose \hat{A} to be equal to the unit operator.
- 2) It looks like as an expression for the Heisenberg operator mean value in the nonrelativistic picture when the operator acts on the wave function taken at the initial moment of time.
- 3) It has natural property $\frac{\langle \hat{A}(\tau) \rangle}{d\tau} = \langle \frac{\hat{A}(\tau)}{d\tau} \rangle$
- 4) It gives physical time value, equal to zero, when proper time equals to zero.

Averaging of the Heisenberg equation of motion gives

$$\begin{aligned} \langle \hat{t}(\tau) \rangle &= \langle \hat{\varepsilon} \rangle \frac{\tau}{m}, \\ \hat{\mathbf{r}}(\tau) &= \langle \hat{\mathbf{r}} \rangle + \langle \hat{\mathbf{p}} \rangle \frac{\tau}{m}. \end{aligned}$$

We see that physical time goes proportional to the proper time. It is interesting to calculate the dispersion of the physical time for the Gaussian wave packet $a(\mathbf{k}) \sim e^{-\mathbf{k}^2}$ with the $\frac{1}{\alpha} \gg m^2$. Evaluating of the mean values of the $\hat{\varepsilon} = i \frac{\partial}{\partial t}$ and its square gives:

$$\begin{aligned} \langle \hat{\varepsilon} \rangle &= \frac{\int_0^\infty \varepsilon(k) e^{-\alpha k^2} k^2 dk}{\int_0^\infty e^{-\alpha k^2} k^2 dk} \approx \frac{2}{\sqrt{\pi\alpha}} \\ \langle \hat{\varepsilon}^2 \rangle &= \frac{\int_0^\infty \varepsilon^2(k) e^{-\alpha k^2} k^2 dk}{\int_0^\infty e^{-\alpha k^2} k^2 dk} \approx \frac{3}{2\alpha}, \end{aligned}$$

and we come to

$$\frac{\sqrt{\langle \hat{t}^2 \rangle - \langle \hat{t} \rangle^2}}{\langle \hat{t} \rangle} = \frac{\sqrt{\langle \hat{\varepsilon}^2 \rangle - \langle \hat{\varepsilon} \rangle^2}}{\langle \hat{\varepsilon} \rangle} = \sqrt{\frac{3\pi}{8}} - 1.$$

Thus, "particle-clock" is a bad clock when particle localized in the region less than Compton wave length. Let us imagine what is to be when we place this particle (for instance having electric charge) in the electric field. For this aim we consider classical picture do not having straight relation to the quantum picture but, still correctly it reflecting. Let us have cloud of the particles with some initial dispersion of the energy placed in the electric field. The particles begin to accelerate in the electric field and receive the energies much larger than its initial energies, so that the energy dispersion becomes negligible. Similar picture holds for the quantum case so that the "particle-clock" from "bad clock" becomes "Swiss clock".

III. QUANTUM COSMOLOGY

We start from the Einstein action of gravity and the action of a one-component real scalar field:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \left[\frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \right], \quad (7)$$

where R is a scalar curvature and V is a matter potential which includes a possible cosmological constant effectively. We restrict our consideration to the homogeneous and isotropic metric:

$$ds^2 = N^2(\tau) d\tau^2 - a^2(\tau) d\sigma^2. \quad (8)$$

Here the lapse function N represents the general time coordinate transformation freedom. For the restricted metric the total action becomes

$$\begin{aligned} S = \int N(\tau) \left\{ \frac{3}{8\pi G} a \left(\mathcal{K} - \frac{\dot{a}^2}{N^2(\tau)} \right) \right. \\ \left. + \frac{1}{2} a^3 \frac{\dot{\phi}^2}{N^2(\tau)} - a^3 V(\phi) \right\} d\tau, \end{aligned}$$

where \mathcal{K} is the signature of the spatial curvature. This action can be obtained from the following expression by varying over p_a and p_ϕ

$$\begin{aligned} S = \int \left\{ p_\phi \dot{\phi} + p_a \dot{a} + N(\tau) \left(-\frac{3a\mathcal{K}}{8\pi G} - \frac{8\pi G p_a^2}{12a} \right. \right. \\ \left. \left. + \frac{p_\phi^2}{2a^3} + a^3 V(\phi) \right) \right\} d\tau. \end{aligned}$$

Varying on N gives the constraint

$$H = -\frac{3a\mathcal{K}}{8\pi G} - \frac{8\pi G p_a^2}{12a} + \frac{p_\phi^2}{2a^3} + a^3 V(\phi) = 0.$$

After quantization $[\hat{a}, \hat{p}_a] = i$, $[\hat{\phi}, \hat{p}_\phi] = i$ this constraint turns into the DeWitt equation

$$\hat{H}\psi(a, \phi) = 0.$$

Every time someone has scratched hands to modernize DeWitt constrain (as, for instance, in [6]). This immediately implies (as in classical so in quantum cases) existence of some preferred system of reference. Although there are some logically consistent theories implying preferred system of reference, for instance, Logunov relativistic theory of gravity [7], giving adequate description of all the stages of Universe expansion [8], still, we prefer to keep to General Relativity here and do not touch the constraint.

After quantization we should to define operator ordering in Hamiltonian. For the first time we take the given operator:

$$\hat{H} = -\frac{1}{4} \left(\hat{p}_a^2 \frac{1}{a} + \frac{1}{a} \hat{p}_a^2 \right) + \frac{\hat{p}_\phi^2}{2a^3} - \frac{\mathcal{K}a}{2} + a^3 V(\phi). \quad (9)$$

In this equation we use system of units which reduce the number of constants. Let us to consider the wave function in the vicinity $a = 0$. When $a \rightarrow 0$ the terms $a^3 V(\phi)$ and $\mathcal{K}a$ do not influence on the form of the wave function and it satisfy to the $H_0\psi = 0$, where the simplified Hamiltonian equals

$$\hat{H}_0 = -\frac{1}{4} \left(\hat{p}_a^2 \frac{1}{a} + \frac{1}{a} \hat{p}_a^2 \right) + \frac{\hat{p}_\phi^2}{2a^3}. \quad (10)$$

Explicit expression for the wave function is

$$\psi(a, \phi) = a^{1 \pm i|k|} e^{ik\phi}.$$

Exactly as in the case of the Klein-Gordon equation we should choose only positive frequency solutions. Thus the wave packet

$$\psi(a, \phi) = \int c(k) \frac{a^{1-i|k|}}{\sqrt{4\pi|k|}} e^{ik\phi} dk. \quad (11)$$

will be normalized by

$$i \int \left(\left(\frac{1}{a} \frac{\partial \psi}{\partial a} \right) \psi^* - \left(\frac{1}{a} \frac{\partial \psi}{\partial a} \right)^* \psi \right) d\phi = \int c^*(k) c(k) dk = 1. \quad (12)$$

As we can see a variable plays here role of the physical time, whereas τ is proper time. After finding of the evolution of the operators in the in the proper time $\frac{d\hat{A}(\tau)}{d\tau} = i[\hat{H}, \hat{A}]$, we find mean it value:

$$\begin{aligned} \langle A(\tau) \rangle = & i \int \left(\frac{1}{a} \frac{\partial \psi}{\partial a} (\hat{A}(\tau) \psi)^* \right. \\ & \left. - \left(\frac{1}{a} \frac{\partial \psi}{\partial a} \right)^* \hat{A}(\tau) \psi \right) d\phi \Big|_{a=0}. \end{aligned} \quad (13)$$

To find evolution for small τ we again may to use simplified Hamiltonian \hat{H}_0 . Evaluation of the commutators gives

$$\dot{\hat{p}}_\phi(\tau) = 0; \quad \hat{p}_\phi(\tau) = const; \quad (14)$$

$$(\hat{a}^3(\tau))' = -\frac{3}{2}(\hat{p}_a a + a \hat{p}_a); \quad (15)$$

$$(\hat{p}_a a + a \hat{p}_a)' = -6\hat{H}_0. \quad (16)$$

From the equation (16) one may conclude that the third derivative from the $\hat{a}^3(\tau)$ is already equal to zero, so $\hat{a}^3(\tau) = a^3 + \hat{D}\tau + \hat{B}\tau^2$. Using (14), (15), (16) allows to find operator constants \hat{D} and \hat{B} :

$$\begin{aligned} \hat{a}^3(\tau) = & a^3 - \frac{3}{2}(\hat{p}_a a + a \hat{p}_a)\tau \\ & + \left(\frac{9}{8} \left(\frac{1}{a} \hat{p}_a^2 + \hat{p}_a^2 \frac{1}{a} \right) - \frac{9}{4} \frac{\hat{p}_\phi^2}{a^3} \right) \tau^2, \end{aligned} \quad (17)$$

where \hat{p}_a, \hat{p}_ϕ are not Heisenberg operators, but ordinary $\hat{p}_a = -i \frac{\partial}{\partial a}$, $\hat{p}_\phi = -i \frac{\partial}{\partial \phi}$.

Evaluation of the mean value over wave packet gives

$$\langle a^3(\tau) \rangle = 3\tau \int \left(\frac{3}{2|k|} + |k| \right) |c(k)|^2 dk. \quad (18)$$

It similar to the classical evolution given by the equations:

$$\begin{aligned} -\frac{p_a^2}{2a} + \frac{p_\phi^2}{2a^3} &= 0; \\ a' &= -\frac{p_a}{a}; \quad \phi' = \frac{p_\phi}{a^3}; \\ p_a' &= -\frac{p_a^2}{2a^2} + \frac{3p_\phi^2}{2a^4}; \\ p_\phi &= const. \end{aligned} \quad (19)$$

Classical evolution gives $a^3(\tau) = 3|p_\phi|\tau$.

Next interesting quantity is mean value from the $\frac{\hat{p}_\phi^2}{2\hat{a}^3}$ which ruffly is energy density of scalar field. In classics the quantity looks like

$$\frac{p_\phi^2}{2a^3(\tau)} = \frac{|p_\phi|}{6\tau} \quad (20)$$

and goes to infinity when $\tau \rightarrow 0$.

For the quantum case we must to find the action of the operator $\frac{1}{\hat{a}^3(\tau)}$ to the wave function i.e. to solve the equation $\hat{a}^3(\tau)\Theta(\tau, a, k) = a^{1-i|k|}$, and find $\Theta(\tau, a, k) = \frac{1}{\hat{a}^3(\tau)} a^{1-i|k|}$.

Explicit solution of the equation looks like

$$\Theta(\tau, a, k) = \frac{a^{1-i|k|}}{3|k|\tau} - \frac{a^{1-i|k|}}{3|k|\tau} \left(\frac{2ia^3}{9\tau} \right)^{\frac{2i}{3}|k|} e^{\frac{2ia^3}{9\tau}} \Gamma\left(1 - \frac{2i}{3}|k|, \frac{2ia^3}{9\tau}\right) \quad (21)$$

where $\Gamma(a, z) = \int_z^\infty t^{a-1} e^{-t} dt$ is incomplete Gamma function. Finding of the mean value of the $\langle \frac{\hat{p}_\phi^2}{\hat{a}^3(\tau)} \rangle$

reduces to the evaluation of the

$$\langle \frac{\hat{p}_\phi^2}{\hat{a}^3(\tau)} \rangle = i \int ((1 + i|k|)a^{-1+i|k|}\Theta(\tau, a, k) - (1 - i|k|)a^{-1-i|k|}\Theta^*(\tau, a, k))|k||c(k)|^2 dk \Big|_{a \rightarrow 0}. \quad (22)$$

Analysis of the asymptotic of the (21) shows that second item containing the Gamma function gives zero contribution after the integration over k and proceeding to the limit $a \rightarrow 0$ for any normalizable $c(k)$. As a result we have

$$\langle \frac{\hat{p}_\phi^2}{\hat{a}^3(\tau)} \rangle = \frac{1}{3\tau} \int |k||c(k)|^2 dk. \quad (23)$$

From this equation we see, that this mean value is singular as it is in the classical theory.

The way to avoid singularity is to suggest, that the Universe was born not from a point but from a "seed" of "size" a_0 . Then expression for mean value changes to

$$\langle A(\tau) \rangle = i \int \left(\frac{1}{a} \frac{\partial \psi}{\partial a} (\hat{A}(\tau)\psi)^* - \left(\frac{1}{a} \frac{\partial \psi}{\partial a} \right)^* \hat{A}(\tau)\psi \right) d\phi \Big|_{a=a_0}.$$

This lead to other question about underling theory giving

size of the seed.

Now we find solution for the Hamiltonian, containing the the cosmological constant V_0 :

$$\hat{H} = -\frac{1}{4} \left(\hat{p}_a^2 \frac{1}{a} + \frac{1}{a} \hat{p}_a^2 \right) + \frac{\hat{p}_\phi^2}{2a^3} + a^3 V_0. \quad (24)$$

Evaluating commutators in a usual way we have

$$(\hat{a}^3(\tau))' = -\frac{3}{2}(\hat{p}_a a + a \hat{p}_a), \quad (25)$$

$$(\hat{p}_a a + a \hat{p}_a)' = 6\hat{H}_0 - 6Va^3, \quad (26)$$

$$6(\hat{H}_0 - Va^3)' = 18V(\hat{p}_a a + a \hat{p}_a). \quad (27)$$

Form the equations (26) and (27) it follows that $(\hat{p}_a a + a \hat{p}_a)'' = 18V_0(\hat{p}_a a + a \hat{p}_a)$ and $(\hat{p}_a a + a \hat{p}_a) = \hat{D} \sinh(\sqrt{18V_0}\tau) + \hat{B} \cosh(\sqrt{18V_0}\tau)$, where \hat{D} and \hat{B} are some operators not depending on τ . Finally arrive to

$$\hat{a}^3(\tau) = a^3 - \frac{3}{2} \left((\hat{p}_a a + a \hat{p}_a) \frac{\sinh(\tau\sqrt{18V})}{3\sqrt{2V}} + \sqrt{\frac{2}{V}} (\hat{H}_0 - Va^3)(\cosh(\tau\sqrt{18V}) - 1) \right). \quad (28)$$

Similar solutions for Heisenberg operators, containing sinh and cosh were obtained in [6, 9]. The operator given by (28) is the local one. Thus to evaluate it mean value according to our rule (13) we need to know the wave function only in the vicinity of $a = 0$, so we can use the function (11), which is limiting value of the exact wave

function at $a \rightarrow 0$. Mean values take the form

$$\begin{aligned} \langle \hat{a}^3(\tau) \rangle &= \frac{\sinh(3\sqrt{2}\sqrt{V}\tau)}{\sqrt{2V}} \int \frac{(3 + 2k^2)}{2|k|} |c(k)|^2 dk \\ \langle \hat{a}^6(\tau) \rangle &= \frac{(\sinh(3\sqrt{2}\sqrt{V}\tau))^2}{2V} \int k^2 |c(k)|^2 dk. \end{aligned}$$

This shows, that dispersion $\frac{\sqrt{\langle \hat{a}^6 \rangle - \langle \hat{a}^3 \rangle^2}}{\langle \hat{a}^3 \rangle}$ is not depends on τ , exactly as in the case of the free relativistic parti-

cle. Thus in this model evolution of the Universe remains quantum all the time. This is because we do not introduce here some fundamental length. Such a fundamental length appears if we take $V(\phi) = \frac{m^2 \phi^2}{2}$. One may suggest that expanding of the Universe before the Compton wave length $1/m$ is quantum, and when $\langle a(\tau) \rangle$ becomes greater than $1/m$ the expansion can be described classically. To see this explicitly we must find Heisenberg solutions with the above potential, but it is very difficult problem.

IV. OPERATOR ORDERING AND THE UNIVERSE REST MASS

In the previous consideration we use some particular operator ordering in the equation (10). In the general case we may write the Hamiltonian in the form

$$\hat{H}_0 = -\frac{1+M^2}{4} \left(\hat{p}_a^2 \frac{1}{a} + \frac{1}{a} \hat{p}_a^2 - \frac{2M^2}{1+M^2} \hat{p}_a \frac{1}{a} \hat{p}_a \right) + \frac{\hat{p}_\phi^2}{2a^3}. \quad (29)$$

To be consistent with the normalization procedure (12) parameter M^2 must be positive. The wave function satisfying to the $H_0 \psi = 0$ is

$$\psi(a, \phi) = a^{1-i\sqrt{M^2+k^2}} e^{ik\phi}. \quad (30)$$

We see that M plays the role of the "rest mass" of the Universe. The equations (25), (26), (27) holds also for the $M \neq 0$ with the corresponding H_0 . It allows us to evaluate mean the values:

$$\begin{aligned} \langle \hat{a}^3(\tau) \rangle &= \frac{\sinh(3\sqrt{2}\sqrt{V}\tau)}{\sqrt{2V}} \int \frac{(3+2k^2+2M^2)}{2\sqrt{k^2+M^2}} |c(k)|^2 dk, \\ \langle \hat{a}^6(\tau) \rangle &= \frac{(\sinh(3\sqrt{2}\sqrt{V}\tau))^2}{2V} \int (k^2+M^2) |c(k)|^2 dk. \end{aligned}$$

Taking into account that $\int |c(k)|^2 dk = 1$ we can conclude that if the Universe have rest mass much greater than the Plank mass (equal to unity in our units) and the character value of the wave vector k , then dispersion of the \hat{a}^3 will be negligible, exactly as in the case of the nonrelativistic particle, for which dispersion of physical time is negligible.

V. CONCLUSION

We have considered quantum evolution of the Universe, originated from the some fluctuation of the scalar field (wave packet).

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